DTIC

(2)

RESEARCH AND DEVELOPMENT TECHNICAL REPORT

SLCET-TR-89-2

AD-A208 349

POLARIZATION MATRICES OF LITHIUM TANTALATE

ARTHUR BALLATO

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

APRIL 1989

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) SLCET-TR-89-2	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION US Army Laboratory Command Electronics Tech & Devices Lab 6b. OFFICE SYMBOL (If applicable) SLCET-MA-A	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Electronics Technology and Devices Laboratory ATTN: SLCET-MA-A Fort Monmouth, NJ 07703-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: 1L162705- PROJECT NO.: H94 TASK NO.: K9 WORK UNIT ACCESSION NO.: DA303394

11. TITLE (Include Security Classification)

POLARIZATION MATRICES OF LITHIUM TANTALATE (U)

12. PERSONAL AUTHOR(S)

Arthur Ballato

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Report	FROM Jan 88 TO Jan 89	1989 April	27

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Piezoelectric resonators; piezoelectric transducers; lithium tantalate; acousto-optics, tantalum compounds, lithium compounds, (mjm)
09	01		
17	02		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

In analytical treatments of piezoelectric acoustic transducers, signal processors, and resonators, the electromechanical transduction mechanism is most often expressed in terms of the elements of the piezoelectric [e] or [d] matrices. Molecular interpretations of piezoelectricity, and especially electro-optical applications, usually involve polarization as the preferred variable, and consequently the alternative [a] and [b] matrices are of interest. The elements of these latter sets are calculated for lithium tantalate from measured elastopiezodielectric constants taken from the literature.

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Arthur Ballato	22b. TELEPHONE (Include Area Code) (201) 544-2773 22c. OFFICE SYMBOL SLCET-MA-A

DD Form 1473, JUN 86 — Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

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INTRODUCTION

Electromechanical transduction taking place via the piezoelectric effect is characterized phenomenologically by constitutive equations that relate the elastic and electric variables. These equations take a variety of forms, depending upon the choice of independent and dependent variables; the choice is normally dictated by the application. For example, piezoelectric resonators in the form of thickness mode plates are most easily treated using the isagric elastic stiffnesses [cE], the piezoelectric stress constants [e], and the dielectric permittivities at constant strain [(eps)S].

Various measurement techniques yield values for the elements of a particular coefficient set more directly than those of another. The coefficients appearing in the different equation sets are, however, interrelated, so that once any one complete set is available, all the other sets of elements may be found. The most accurate and precise experimental results to date have been from plate resonator (resonance) and pulse-echo (transit-time) measurements. From the [cE], [e], and [(eps)S] matrices determined therefrom, those matrices representing material properties expressed in the other alternative forms may be calculated.

Electrooptical applications are becoming increasingly important. So also are treatments of piezoelectric and ferroelectric phenomena from the standpoint of molecular interactions. In both of these cases the constitutive equations using polarization as the independent electrical variable, rather than either electric intensity or displacement, assume greater importance than the sets traditionally used for transducer, signal processing, and resonator applications.

In this report we give the complete sets of linear constitutive equations relating elastic and electric fields. For each equation set the numerical values are computed for lithium tantalate, a highly piezoelectric ferroelectric with temperature-compensated orientations, from the measured [cE], [e], and [(eps)S] values of Smith and Welsh (Ref. 1). Coupling to the thermal field is neglected. Rationalized mks units are used throughout.

CONSTITUTIVE EQUATION SETS

Symbols and units for the quantities employed are given in Table 1. In terms of these, six constitutive equation sets are used. Of these, electric intensity, dielectric displacement, and polarization each appear in two sets as an independent variable. The sets are, in compressed matrix notation, as follows. A prime denotes transpose; [I] is the unit matrix.

I. The Piezoelectric Stress Constant Set

$$[T] = [cE]\,[S] - [e]'\,[E] \qquad (1)$$
$$[D] = [e]\,[S] + [(eps)S]\,[E] \qquad (2)$$

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS.

QUANTITY	UNIT	SYMBOL/DEFINITION
Elastic stress	N/m2	[T]
Elastic strain	-----	[S]
Electric intensity	V/m	[E]
Dielectric displacement	C/m2	[D]
Dielectric polarization	C/m2	[P]
Elastic compliance at constant [E], [D], [P]	m2/N	[cE], [cD], [cP]
Elastic stiffness at constant [E], [D], [P]	N/m2	[sE], [sD], [sP]
Dielectric permittivity at constant [T], [S]	F/m	[(eps)T], [(eps)S]
Dielectric constant, relative, at constant [T], [S]	-----	[(Kr)T], [(Kr)S] =[(eps)T]/(eps)o, [(eps)S]/(eps)o
Dielectric impermeability at constant [T], [S]	m/F	[(bet)T], [(bet)S] =[(eps)T]$^{(-1)}$, [(eps)S]$^{(-1)}$
Dielectric impermeability, relative, at constant [T], [S]	-----	[(betr)T], [(betr)S] =[(bet)T]*(eps)o, [(bet)S]*(eps)o =[(Kr)T]$^{(-1)}$, [(Kr)S]$^{(-1)}$
Dielectric susceptibility at constant [T], [S]	F/m	[(chi)T], [(chi)S] =[(Kr)T-I]*(eps)o, [(Kr)S-I]*(eps)o
Dielectric susceptibility, relative, at constant [T], [S]	-----	[(chir)T], [(chir)S] =[(chi)T]/(eps)o, [(chi)S]/(eps)o
Reciprocal dielectric susceptibility at constant [T], [S]	m/F	[(zet)T], [(zet)S] =[(chi)T]$^{(-1)}$, [(chi)S]$^{(-1)}$
Reciprocal dielectric susceptibility, relative, at constant [T], [S]	-----	[(zetr)T], [(zetr)S] =[(zet)T]*(eps)o, [(zet)S]*(eps)o
Piezoelectric stress constant	C/m2	[e]

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS. (continued)

QUANTITY	UNIT	SYMBOL/DEFINITION
Piezoelectric strain coefficient	m/V = C/N	[d]
Piezoelectric stress modulus	N/C = V/m	[h]
Piezoelectric strain constant	m2/C	[g]
Piezoelectric polarization modulus	V/m = N/C	[a]
Piezoelectric polarization constant	m2/C	[b]

Note: Square brackets, sic: [ ], denote matrices.

II. The Piezoelectric Strain Coefficient Set

[S] = [sE] [T] + [d]' [E] (3)

[D] = [d] [T] + [(eps)T] [E] (4)

III. The Piezoelectric Stress Modulus Set

[T] = [cD] [S] - [h]' [D] (5)

[E] = -[h] [S] + [(bet)S] [D] (6)

IV. The Piezoelectric Strain Constant Set

[S] = [sD] [T] + [g]' [D] (7)

[E] = -[g] [T] + [(bet)T] [D] (8)

V. The Piezoelectric Polarization Modulus Set

[T] = [cP] [S] - [a]' [P] (9)

[E] = -[a] [S] + [(zet)S] [P] (10)

VI. The Piezoelectric Polarization Constant Set

[S] = [sP] [T] + [b]' [P] (11)

[E] = -[b] [T] + [(zet)T] [P] (12)

The electric variables are connected by the relation

[D] = (eps)o * [E] + [P] (13)

where (eps)o is the permittivity of free space, defined by

(eps)o * (mu)o * (c) * (c) = 1 ; (14)

(mu)o is the permeability of free space, equal, by definition, to 4 PI * $10^{(-7)}$, and (c) is the velocity of light in vacuo and, also by definition, is equal exactly to 2.99792458 x 10^8 m/s.

From (13) the expressions for the remaining electric variables associated, respectively, with the six equation sets (1) to (12) may be found:

[P] = [e] [S] + [(chi)S] [E] (15)

[P] = [d] [T] + [(chi)T] [E] (16)

[P] = (eps)o * [h] [S] + [I - (eps)o * (bet)S] [D] (17)

[P] = (eps)o * [g] [T] + [I - (eps)o * (bet)T] [D] (18)

$$[D] = -(eps)o * [a]\,[S] + [I + (eps)o * (zet)S]\,[P] \tag{19}$$

$$[D] = -(eps)o * [b]\,[T] + [I + (eps)o * (zet)T]\,[P] \tag{20}$$

RELATIONS AMONG MATERIAL CONSTANTS

The material constants are interrelated by the following formulas:

$$[cX]\,[sX] = [(eps)Y]\,[(bet)Y] = [I] \tag{21}$$

$$[(chi)Y]\,[(zet)Y] = [(Kr)Y - (chir)Y] = [I] \tag{22}$$

In (21) and (22), X = E, D, or P and Y = T or S.

$$\begin{aligned}[cD] - [cE] &= [h]'\,[e] = [e]'\,[h] \\ &= [h]'\,[(eps)S]\,[h] = [e]'\,[(bet)S]\,[e] \\ &= [a]'\,[e - h * (eps)o] = [e - h * (eps)o]'\,[a]\end{aligned} \tag{23}$$

$$\begin{aligned}[cP] - [cD] &= [h]'\,[a] * (eps)o = [a]'\,[h] * (eps)o \\ &= [h]'\,[(eps)S]\,[(zet)S]\,[h] * (eps)o \\ &= [a]'\,[(bet)S]\,[(chi)S]\,[a] * (eps)o \\ &= [a - h]'\,[e] = [e]'\,[a - h]\end{aligned} \tag{24}$$

$$\begin{aligned}[cP] - [cE] &= [a]'\,[e] = [e]'\,[a] \\ &= [a]'\,[(chi)S]\,[a] = [e]'\,[(zet)S]\,[e] \\ &= [h]'\,[e + a * (eps)o] = [e + a * (eps)o]'\,[h]\end{aligned} \tag{25}$$

$$\begin{aligned}[sE] - [sD] &= [d]'\,[g] = [g]'\,[d] \\ &= [d]'\,[(bet)t]\,[d] = [g]'\,[(eps)t]\,[g] \\ &= [b]'\,[d - g * (eps)o] = [d - g * (eps)o]'\,[b]\end{aligned} \tag{26}$$

$$\begin{aligned}[sD] - [sP] &= [b]'\,[g] * (eps)o = [g]'\,[b] * (eps)o \\ &= [g]'\,[(eps)T]\,[(zet)T]\,[g] * (eps)o \\ &= [b]'\,[(bet)T]\,[(chi)T]\,[b] * (eps)o \\ &= [b - g]'\,[d] = [d]'\,[b - g]\end{aligned} \tag{27}$$

$$\begin{aligned}[sE] - [sP] &= [b]'\,[d] = [d]'\,[b] \\ &= [b]'\,[(chi)T]\,[b] = [d]'\,[(zet)T]\,[d] \\ &= [g]'\,[d + b * (eps)o] = [d + b * (eps)o]'\,[g]\end{aligned} \tag{28}$$

[(zet)S] - [(zet)T] = [b] [a]' = [a] [b]'

= [b] [cP] [b]' = [a] [sP] [a]' (29)

[(chi)T] - [(chi)S] = [(eps)T] - [(eps)S]

= [e] [d]' = [d] [e]'

= [d] [cE] [d]' = [e] [sE] [e]' (30)

[(bet)S] - [(bet)T] = [h] [g]' = [g] [h]'

= [g] [cD] [g]' = [h] [sD] [h]' (31)

[e] = [d] [cE] = [(eps)S] [h] = [(chi)S] [a] (32)

[d] = [e] [sE] = [(eps)T] [g] = [(chi)T] [b] (33)

[h] = [g] [cD] = [(bet)S] [e] = [(chi)S] [(bet)S] [a]

= [I - (bet)S * (eps)o] [a] (34)

[g] = [h] [sD] = [(bet)T] [d] = [(chi)T] [(bet)T] [b]

= [I - (bet)T * (eps)o] [b] (35)

[a] = [b] [cP] = [(zet)S] [e] = [(eps)S] [(zet)S] [h]

= [I + (zet)S * (eps)o] [h] (36)

[b] = [a] [sP] = [(zet)T] [d] = [(eps)T] [(zet)T] [g]

= [I + (zet)T * (eps)o] [g] (37)

Some alternative relations are the following:

[a - h] = [(zet)S] [h] * (eps)o

= [(bet)S] [a] * (eps)o (38)

[b - g] = [(zet)T] [g] * (eps)o

= [(bet)T] [b] * (eps)o (39)

[e + a * (eps)o] = [(eps)S] [a] (40)

[d + b * (eps)o] = [(eps)T] [b] (41)

[e - h * (eps)o] = [(chi)S] [h] (42)

[d - g * (eps)o] = [(chi)T] [g] (43)

Equations (21) to (43) result from equating like dependent variables in pairs selected from equations (1) to (12) and (15) to (20). Each pair yields one equation in three variables, one mechanical and two electrical, or vice versa. Two other equations exist, again from (1) to (12) and (15) to (20), that contain the same three variables found in each paired equation. One of these auxiliary equations is used to eliminate one of the two variables of the same kind; the result is one equation in two variables, one electrical and one mechanical. These are now independent variables, so the coefficients must vanish; two relations between the material coefficients result. As an example, (3) and (7) both have [S] as dependent variable. Equating them produces one relation in [T], [E], and [D]; one of the electrical variables must be eliminated. This is done by using either (4) or (8); each contains the same three variables. If (8) is used to eliminate [E], one obtains [sE - d' g - sD] [T] = [d' (bet)T - g'] [D]. Therefore, [sE] - [sD] = [d]' [g] and [g] = [(bet)T] [d]. Use of (4) instead of (8) leads to the equations [sE] - [sD] = [g]' [d] and [d] = [(eps)T] [g]. There are 36 pairs, six each equating [S] and [T], and eight each equating [E], [D], and [P]. The 72 relations contain many redundancies. Relations between the elastic, piezoelectric, and dielectric constants are shown schematically in Tables 2 and 3.

CALCULATION SEQUENCE

Using as input [cE], [e], and [(eps)S], one may compute the remaining quantities in a variety of ways. The following sequence is typical:

$$[sE] = [cE]^{(-1)} \tag{44}$$

$$[(bet)S] = [(eps)S]^{(-1)} \tag{45}$$

$$[d] = [e]\ [sE] \tag{46}$$

$$[h] = [(bet)S]\ [e] \tag{47}$$

$$[(eps)T] - [(eps)S] = [e]\ [d]' \tag{48}$$

$$[(eps)T] = [(eps)S] + [e]\ [d]' \tag{49}$$

$$[(bet)T] = [(eps)T]^{(-1)} \tag{50}$$

$$[cD] - [cE] = [e]'\ [h] \tag{51}$$

$$[cD] = [cE] + [e]'\ [h] \tag{52}$$

$$[g] = [(bet)T]\ [d] \tag{53}$$

$$[sE] - [sD] = [d]'\ [g] \tag{54}$$

$$[sD] = [sE] - [d]'\ [g] \tag{55}$$

$$[(betr)S] = [(bet)S] * (eps)o \tag{56}$$

$$[(zetr)S] = [(betr)S]\ (I - (betr)S]^{(-1)} \tag{57}$$

$$[(zet)S] = [(zetr)]\ /\ (eps)o \tag{58}$$

TABLE 2. RELATIONS AMONG MATERIAL CONSTANTS.

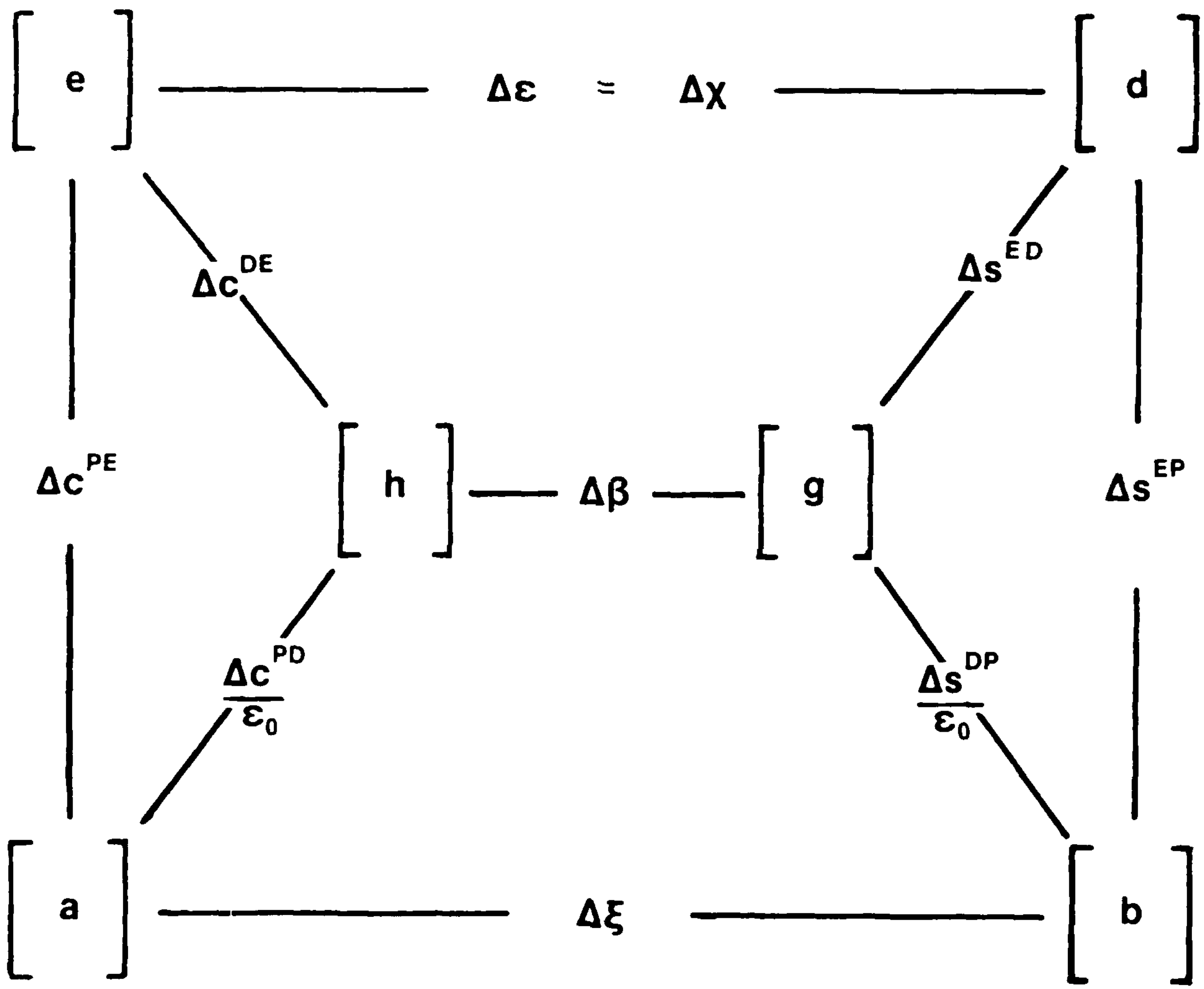
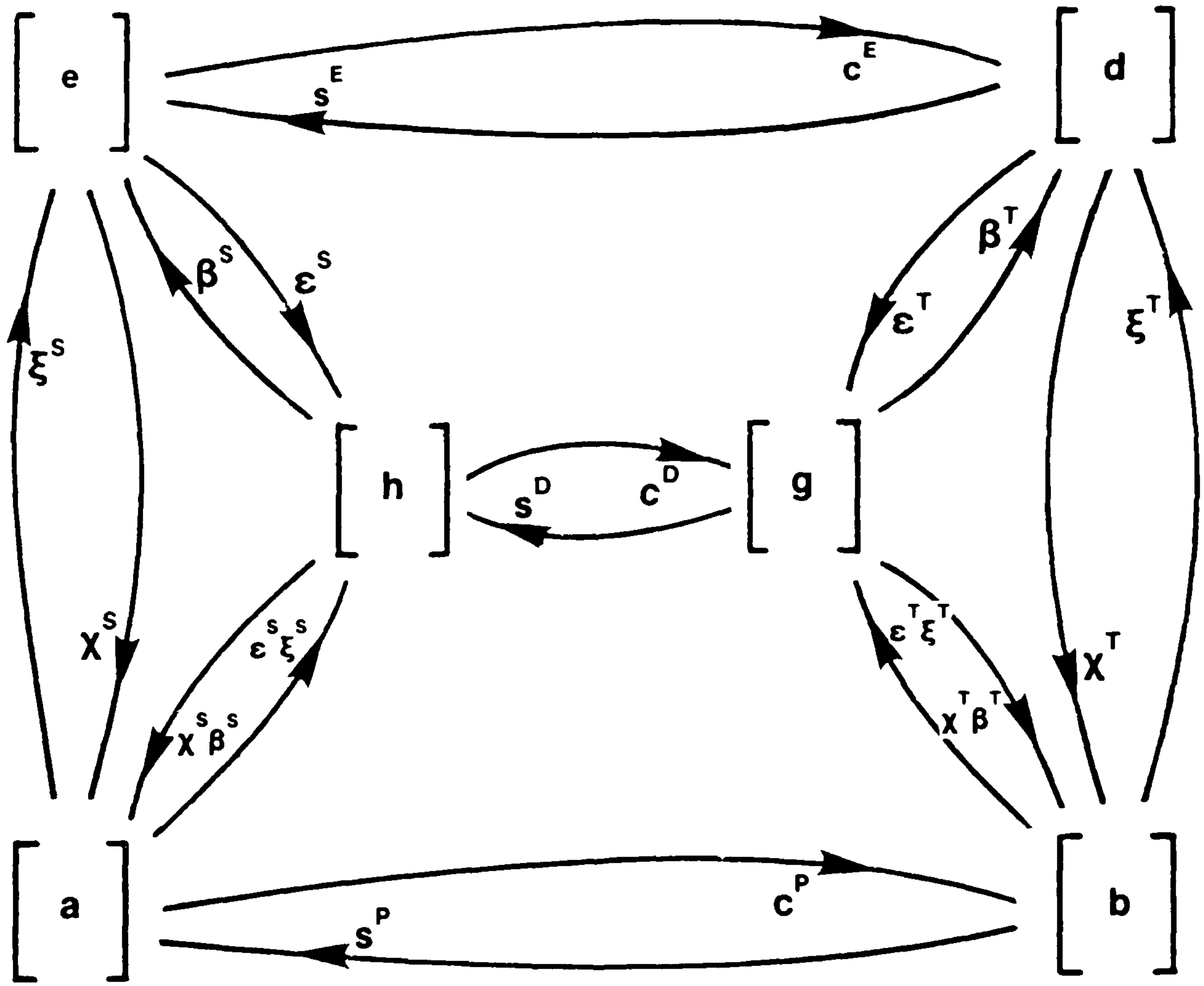


TABLE 3. FURTHER RELATIONS AMONG MATERIAL CONSTANTS.



[(betr)T] = [(bet)T] * (eps)o (59)

[(zetr)T] = [(betrT] [I - (betr)T] $^{(-1)}$ (60)

[(zet)T = [(zetr)T] / (eps)o (61)

[(chi)S] = [(zet)S] $^{(-1)}$ (62)

[(chi)T] = [(zet)T] $^{(-1)}$ (63)

[a] = [(zet)S] [e] (64)

[b] = [(zet)T] [d] (65)

[cP] - [cE] = [e]' [a] (66)

[cP] = [cE] + [e]' [a] (67)

[cP] - [cD] = [a]' [h] * (eps)o (68)

[sE] - [sP] = [d]' [b] (69)

[sP] = [sE] - [d]' [b] (70)

[sD] - [sP] = [g]' [b] * (eps)o (71)

[(bet)S] - [(bet)T] = [h] [g]' (72)

[(chi)T] - [(chi)S] = [(eps)T] - [(eps)S] (73)

[(zet)S] - [(zet)T] = [a] [b]' (74)

A number of these relations are used as checks. For example, [(bet)S] and [(bet)T] are known from (45) and (50), but the difference is recomputed in (72).

EXPLICIT FORMULAS FOR POINT GROUP 3m

Elastic:

The 6x6 elastic constant portions of Tables 4 and 5 partition into 4x4 and 2x2 submatrices. The 4x4 elastic stiffness and compliance submatrices are interrelated by formulas (75) to (93), taken from Cady (Ref. 2):

A = s33 * (s11 + s12) - 2 * s13 * s13 (75)

B = s44 * (s11 - s12) - 2 * s14 * s14 (76)

2 * c11 = s33 / A + s44 / B (77)

2 * c12 = s33 / A - s44 / B (78)

c13 = - s13 / A ; c14 = - s14 / B (79a), (79b) (79)

TABLE 4. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 3m: THE [e], [h], AND [a] SETS.

11	12	13	14	00	00	]	00	-22	31	
12	11	13	-14	00	00	]	00	22	31	cE] e' ----]------ e](eps)S
13	13	33	00	00	00	]	00	00	33	
14	-14	00	44	00	00	]	00	15	00	
00	00	00	00	44	14	]	15	00	00	cD] h' ----]------ h](bet)S
00	00	00	00	14	66	]	-22	00	00	
00	00	00	00	15	-22	]	11	00	00	
-22	22	00	15	00	00	]	00	11	00	cP] a' ----]------ a](zet)S
31	31	33	00	00	00	]	00	00	33	

66 = (11 - 12) / 2

Matrix entries show only subscripts.

TABLE 5. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 3m: THE [d], [g], AND [b] SETS.

11	12	13	14	00	00	]	00	-22	31	
12	11	13	-14	00	00	]	00	22	31	sE] d' ----]------ d](eps)T
13	13	33	00	00	00	]	00	00	33	
14	-14	00	44	00	00	]	00	15	00	
00	00	00	00	44	14*2	]	15	00	00	sD] g' ----]------ g](bet)T
00	00	00	00	14*2	66	]	-22*2	00	00	
00	00	00	00	15	-22*2	]	11	00	00	
-22	22	00	15	00	00	]	00	11	00	sP] b' ----]------ b](zet)T
31	31	33	00	00	00	]	00	00	33	

66 = (11 - 12) * 2

Matrix elements show only subscripts.

$$c33 = (s11 + s12) / A \qquad (80)$$

$$c44 = (s11 - s12) / B \qquad (81)$$

$$c66 = (c11 - c12) / 2 = s44 / (2 * B) \qquad (82)$$

$$K = c33 * (c11 + c12) - 2 * c13 * c13 \qquad (83)$$

$$L = c44 * (c11 - c12) - 2 * c14 * c14 \qquad (84)$$

$$2 * s11 = c33 / K + c44 / L \qquad (85)$$

$$2 * s12 = c33 / K - c44 / L \qquad (86)$$

$$s13 = - c13 / K \;;\; s14 = - c14 / L \qquad (87a),\ (87b) \qquad (87)$$

$$s33 = (c11 + c12) / K \qquad (88)$$

$$s44 = (c11 - c12) / L \qquad (89)$$

$$s66 = (s11 - s12) = 2 * c44 / L \qquad (90)$$

$$\det\ (4x4)\ [s] = A * B \qquad (91)$$

$$\det\ (4x4)\ [c] = K * L \qquad (92)$$

$$A * K = B * L = A * B * K * L = 1 \qquad (93)$$

Formulas (75) to (93) hold for each set of constant electrical conditions: either E, D, or P constant.

$$[cD] - [cE] = [del\ cDE] = [e]'\ [h] = [h]'\ [e] \qquad (23)$$

$$del\ cDE11 = + e22\ h22 + e31\ h31 \qquad (94)$$

$$del\ cDE12 = - e22\ h22 + e31\ h31 \qquad (95)$$

$$del\ cDE13 = + e31\ h33 = + h31\ e33 \qquad (96)$$

$$del\ cDE14 = - e22\ h15 = - h22\ e15 \qquad (97)$$

$$del\ cDE33 = + e33\ h33 \qquad (98)$$

$$del\ cDE44 = + e15\ h15 \qquad (99)$$

$$del\ cDE66 = + e22\ h22 \qquad (100)$$

$$[cP] - [cD] = [del\ cPD] = [a]'\ [h] * (eps)o$$

$$= [h]'\ [a] * (eps)o \qquad (24)$$

$$del\ cPD11 = (+ a22\ h22 + a31\ h31) * (eps)o \qquad (101)$$

$$del\ cPD12 = (- a22\ h22 + a31\ h31) * (eps)o \qquad (102)$$

$$\text{del cPD13} = (+ a31\ h33) * (\text{eps})o$$

$$= (+ h31\ a33) * (\text{eps})o \quad (103)$$

$$\text{del cPD14} = (- a22\ h15) * (\text{eps})o$$

$$= (- h22\ a15) * (\text{eps})o \quad (104)$$

$$\text{del cPD33} = (+ a33\ h33) * (\text{eps})o \quad (105)$$

$$\text{del cPD44} = (+ a15\ h15) * (\text{eps})o \quad (106)$$

$$\text{del cPD66} = (+ a22\ h22) * (\text{eps})o \quad (107)$$

$$[cP] - [cE] = [\text{del cPE}] = [e]'\ [a] = [a]'\ [e] \quad (25)$$

$$\text{del cPE11} = + e22\ a22 + e31\ a31 \quad (108)$$

$$\text{del cPE12} = - e22\ a22 + e31\ a31 \quad (109)$$

$$\text{del cPE13} = + e31\ a33 = + a31\ e33 \quad (110)$$

$$\text{del cPE14} = - e22\ a15 = - a22\ e15 \quad (111)$$

$$\text{del cPE33} = + e33\ a33 \quad (112)$$

$$\text{del cPE44} = + e15\ a15 \quad (113)$$

$$\text{del cPE66} = + e22\ a22 \quad (114)$$

From the del c13 entries we have the ratios

$$e31 / e33 = h31 / h33 = a31 / a33. \quad (115)$$

From the del c14 entries we have the further ratios

$$e15 / e22 = h15 / h22 = a15 / a22. \quad (116)$$

$$[sE] - [sD] = [\text{del sED}] = [d]'\ [g] = [g]'\ [d] \quad (26)$$

$$\text{del sED11} = + d22\ g22 + d31\ g31 \quad (117)$$

$$\text{del sED12} = - d22\ g22 + d31\ g31 \quad (118)$$

$$\text{del sED13} = + d31\ g33 = + g31\ d33 \quad (119)$$

$$\text{del sED14} = - d22\ g15 = - g22\ d15 \quad (120)$$

$$\text{del sED33} = + d33\ g33 \quad (121)$$

$$\text{del sED44} = + d15\ g15 \quad (122)$$

$$\text{del sED66} = + d22\ g22 * 4 \quad (123)$$

$$[sD] - [sP] = [g]'\ [b] * (eps)o$$

$$= [b]'\ [g] * (eps)o \tag{27}$$

$$\text{del } sDP11 = (+ g22\ b22 + g31\ b31) * (eps)o \tag{124}$$

$$\text{del } sDP12 = (- g22\ b22 + g31\ b31) * (eps)o \tag{125}$$

$$\text{del } sDP13 = (+ g31\ b33) * (eps)o$$

$$= (+ b31\ g33) * (eps)o \tag{126}$$

$$\text{del } sDP14 = (- g22\ b15) * (eps)o$$

$$= (- b22\ g15) * (eps)o \tag{127}$$

$$\text{del } sDP33 = (+ g33\ b33) * (eps)o \tag{128}$$

$$\text{del } sDP44 = (+ g15\ b15) * (eps)o \tag{129}$$

$$\text{del } sDP66 = (+ g22\ b22) * 4 * (eps)o \tag{130}$$

$$[sE] - [sP] = [\text{del } sEP] = [b]'\ [d] = [d]'\ [b] \tag{28}$$

$$\text{del } sEP11 = + d22\ b22 + d31\ b31 \tag{131}$$

$$\text{del } sEP12 = - d22\ b22 + d31\ b31 \tag{132}$$

$$\text{del } sEP13 = + d31\ b33 = + b31\ d33 \tag{133}$$

$$\text{del } sEP14 = - d22\ b15 = - b22\ d15 \tag{134}$$

$$\text{del } sEP33 = + d33\ b33 \tag{135}$$

$$\text{del } sEP44 = + d15\ b15 \tag{136}$$

$$\text{del } sEP66 = + d22\ b22 * 4 \tag{137}$$

From the del s13 entries we have the ratios

$$d31 / d33 = g31 / g33 = b31 / b33. \tag{138}$$

From the del s14 entries we have the further ratios

$$d15 / d22 = g15 / g22 = b15 / b22. \tag{139}$$

Piezoelectric:

$$[d] = [e]\ [sE] \tag{33}$$

$$d15 = + e15\ sE44 - e22\ sE14 * 2 \tag{140}$$

$$d22 = + e22\ (sE11 - sE12) - e15\ sE14 \tag{141}$$

$$d31 = +\, e31\, (sE11 + sE12) + e33\, sE13 \tag{142}$$

$$d33 = +\, e33\, sE33 + e13\, sE13 * 2 \tag{143}$$

$$[h] = [(bet)S]\, [e] \tag{34}$$

$$h15 = (bet)S11\, e15 \tag{144}$$

$$h22 = (bet)S11\, e22 \tag{145}$$

$$h31 = (bet)S33\, e31 \tag{146}$$

$$h33 = (bet)S33\, e33 \tag{147}$$

$$[g] = [(bet)T]\, [d] \tag{35}$$

$$g15 = (bet)T11\, d15 \tag{148}$$

$$g22 = (bet)T11\, d22 \tag{149}$$

$$g31 = (bet)T33\, d31 \tag{150}$$

$$g33 = (bet)T33\, d33 \tag{151}$$

$$[a] = [(zet)S]\, [e] \tag{36}$$

$$a15 = (zet)S11\, e15 \tag{152}$$

$$a22 = (zet)S11\, e22 \tag{153}$$

$$a31 = (zet)S33\, e31 \tag{154}$$

$$a33 = (zet)S33\, e31 \tag{155}$$

$$[b] = [(zet)T]\, [d] \tag{37}$$

$$b15 = (zet)T11\, d15 \tag{156}$$

$$b22 = (zet)T11\, d22 \tag{157}$$

$$b31 = (zet)T33\, d31 \tag{158}$$

$$b33 = (zet)T33\, d33 \tag{159}$$

Dielectric:

$$[(bet)Y] = [(eps)Y]^{(-1)} \tag{21}$$

$$(bet)Y11 = 1\, /\, (eps)Y11 \tag{160}$$

$$(bet)Y33 = 1\, /\, (eps)Y33 \tag{161}$$

```
[(zetr)Y] = [(betr)Y] [I - (betr)Y]^(-1)                    (162)
(zet)Y11 = 1 / ((eps)Y11 - (eps)o)                          (163)
(zet)Y33 = 1 / ((eps)Y33 - (eps)o)                          (164)

[(eps)T - (eps)S] = [del (eps)] = [e] [d]' =
    [(chi)T - (chi)S] = [del (chi)] = [d] [e]'              (30)
del (eps)11 = del (chi)11 =  + e15 d15 + e22 d22 * 2        (165)
del (eps)33 = del (chi)33 =  + e33 d33 + e31 d31 * 2        (166)

[(bet)S - (bet)T] = [h] [g]' = [g] [h]'                     (31)
             del (bet)11 = + h15 g15 + h22 g22 * 2          (167)
             del (bet)33 = + h33 g33 + h31 g31 * 2          (168)
[(zet)S - (zet)T] = [del (zet)] = [a] [b]' = [b] [a]'       (169)
             del (zet)11 = + a15 b15 + a22 b22 * 2          (170)
             del (zet)33 = + a33 b33 + a31 b31 * 2          (171)
```

INPUT VALUES FOR LI TA 03

The values measured by Smith and Welsh (Ref.1) using the pulse-echo (transit-time) technique are as follows:

TABLE 6. ISAGRIC ELASTIC STIFFNESSES.

cE11	cE12	cE13	cE14	cE33	cE44	cE66
229.8	44.0	81.2	-10.4	279.8	96.8	92.9

Units: $10^{(9)}$ N/m2.

TABLE 7. PIEZOELECTRIC STRESS CONSTANTS.

e15	e22	e31	e33
2.72	1.67	-0.38	1.09

Units: C/m2.

TABLE 8. DIELECTRIC PERMITTIVITIES AT CONSTANT STRAIN.

(eps)S11	(eps)S33
377.	379.

Units: $10^{(-12)}$ F/m.

OUTPUT VALUES FOR LI TA 03

The input values from Tables 6, 7, and 8 were used to compute the remaining elastic, piezoelectric, and dielectric quantities for lithium tantalate in the manner discussed in prior sections of this report. The results are given in Tables 9 to 16.

TABLE 9. ELASTIC STIFFNESSES.

	cE	cD	cP	del cDE	del cPE	del cPD
11	229.8	237.6	237.8	7.78	7.976	0.187
12	44.0	37.0	36.8	-7.02	-7.19	-0.169
13	81.2	80.1	80.1	-1.09	-1.12	-0.0261
14	-10.4	-22.4	-22.7	-12.0	-12.3	-0.291
33	279.8	282.9	283.0	3.13	3.21	0.0750
44	96.8	116.4	116.9	19.6	20.1	0.472
66	92.9	100.3	100.5	7.40	7.58	0.178

Units: $10^{(9)}$ N/m2.

TABLE 10. ELASTIC COMPLIANCES.

	sE	sD	sP	del sED	del sEP	del sDP
11	4.93	4.79	4.78	0.143	0.146	0.00283
12	-0.518	-0.424	-0.422	-0.0945	-0.0962	-0.00169
13	-1.28	-1.24	-1.23	0.0450	-0.0461	-0.00105
14	0.585	1.00	1.01	-0.419	-0.427	-0.00798
33	4.32	4.23	4.23	0.0832	0.0852	0.00195
44	10.46	8.98	8.95	1.48	1.51	0.0282
66	10.90	10.42	10.41	0.475	0.484	0.00905

Units: $10^{(-12)}$ m2/N.

TABLE 11. PIEZOELECTRIC [e], [h], AND [a] VALUES.

	e	h	a
15	2.72	7.21	7.39
22	1.67	4.43	4.54
31	-0.38	-1.00	-1.03
33	1.09	2.88	2.94

Units: e: C / m2; h and a: $10^{(9)}$ V/m.

TABLE 12. PIEZOELECTRIC [d], [g], AND [b] VALUES.

	d	g	b
15	26.5	55.9	56.9
22	7.51	15.8	16.1
31	-3.07	-7.93	-8.11
33	5.68	14.7	15.0

Units: d: $10^{(-12)}$ m / V; g and b: $10^{(-3)}$ m2/C.

TABLE 13. DIELECTRIC (eps) VALUES.

	(eps)S	(eps)T	del (eps)TS
11	377.	474.	97.1
33	247.0	254.6	8.52

Units: $10^{(-12)}$ F/m.

del (eps)TS = del (chi)TS

TABLE 14. DIELECTRIC (chi) VALUES.

	(chi)S	(chi)T	del (chi)TS
11	368.	465.7	97.1
33	370.	479.	8.52

Units: $10^{(-12)}$ F/m.

del (chi)TS = del (eps)TS

TABLE 15. DIELECTRIC (bet) VALUES.

	(bet)S	(bet)T	del (bet)TS
11	2.65	2.11	-0.543
33	2.64	2.58	-0.0580

Units: $10^{(9)}$ m/F.

TABLE 16. DIELECTRIC (zet) VALUES.

	(zet)S	(zet)T	del (zet)TS
11	2.72	2.15	-0.567
33	2.70	2.64	-0.0608

Units: $10^{(9)}$ m/F.

CONCLUSIONS

This report provides formulas interrelating the coefficients that appear in the several alternative sets of constitutive equations involving the elastic, piezoelectric, and dielectric properties of crystals. These are then specialized for crystals of class 3m; using measured values reported for lithium tantalate, numerical values of the elements of the polarization matrices are calculated.

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